Physics Study Material

ATOMS

Updated for 2026 – 2027 Batch

Name: _____

Date: _____

1. What is this Chapter About?

This chapter takes you inside the **atom** – the fundamental building block of all matter – and answers one of the oldest questions in physics: *"What does an atom actually look like on the inside?"*
You will trace the historical journey of atomic models, starting from J.J. Thomson's **"plum-pudding" model**, moving through Ernest Rutherford's landmark **α-particle scattering experiment** (which revealed the nucleus), and finally arriving at Niels Bohr's **quantised model of the hydrogen atom** – the first model to successfully explain why atoms are stable and why they emit light only at specific, sharp wavelengths (spectral lines).
In short, this chapter bridges **classical physics** (which failed to explain atomic stability) with **early quantum theory** (which succeeded), making it one of the most conceptually important chapters in your Class 12 syllabus.

2. What Will You Learn?

1. Thomson's model of the atom and its limitations
2. Rutherford's α-scattering experiment & discovery of the nucleus
3. Distance of closest approach & impact parameter
4. Rutherford's atomic model and its drawback (stability problem)
5. Bohr's postulates and the Bohr model of the hydrogen atom
6. Energy levels, radius, and velocity of electron orbits
7. Hydrogen spectrum – Lyman, Balmer, Paschen, Brackett, Pfund series
8. de Broglie's explanation for Bohr's quantisation condition
9. Limitations of the Bohr model

3. Why is it Useful to You?

- **Conceptual value:** First real introduction to **quantum ideas** (quantised energy, angular momentum) before you study modern physics in higher studies.
- **Foundation for Chapter 13 (Nuclei):** Concepts of energy levels and quantisation carry forward directly.
- **Competitive exams:** Core topic for **KCET, NEET, JEE** – questions on Bohr's model, spectral series, and energy calculations appear almost every year.
- **Real-world link:** Explains atomic spectra, lasers, and spectroscopy used in astronomy and chemistry.

4. PUC Exam Weightage (DPUE Blueprint)

Karnataka II PUC Physics theory paper = 70 marks.

As per the DPUE (Department of Pre-University Education) blueprint, **Atoms** typically carries **4–6 marks** on its own, usually through a combination of:

- 1 mark (VSA) question
- 2 mark short-answer question
- Occasional numerical problem (linked with Nuclei)

Together with **Dual Nature of Radiation & Matter** and **Nuclei**, this "Modern Physics" cluster contributes **15–17 marks** – one of the highest-weightage clusters in the paper, alongside Current Electricity and Optics.

Chapter	Marks
Dual Nature of Matter	6–7
Atoms	**4–6**
Nuclei	5–8
Total (Modern Physics)	**15–17**

Note: Exact split varies slightly year to year as per official DPUE blueprint.

DALTON'S ATOMIC THEORY (1808)

Even before electrons, protons, or nuclei were discovered, the Greek philosopher **Democritus** (˜400 BC) had proposed – purely on logical grounds, with no experiment – that matter is made of indivisible particles called *"atomos."* It took over 2000 years for a scientist to turn this philosophical idea into a testable scientific theory. In 1808, the English chemist **John Dalton** did exactly that, using the experimentally-verified **laws of chemical combination** (conservation of mass, definite proportions, and multiple proportions) as his evidence base.

Postulates of Dalton's Atomic Theory

1. All matter is made up of extremely small, indivisible particles called **atoms**.
2. Atoms of a given element are identical to one another in mass, size, and all other chemical properties; atoms of different elements differ in mass and properties.
3. Atoms can neither be created nor destroyed in a chemical reaction *(explains the Law of Conservation of Mass).*
4. Atoms of different elements combine in simple, fixed, whole-number ratios to form **compound atoms** (molecules) *(explains the Law of Definite Proportions).*
5. Atoms of the same elements can combine in more than one whole-number ratio to form two or more different compounds *(explains the Law of Multiple Proportions).*
6. The atom is the smallest particle that takes part in a chemical reaction.

Limitations of Dalton's Theory

- **Atoms are NOT indivisible:** Discovery of the electron (J.J. Thomson, 1897) and later the proton and neutron showed atoms have internal structure.
- **Atoms of the same element are NOT always identical:** Discovery of **isotopes** (e.g. ^{35}Cl and ^{37}Cl) showed atoms of the same element can have different masses.
- **Atoms of different elements are NOT always different: Isobars** (e.g. $^{40}_{18}$Ar and $^{40}_{20}$Ca) are atoms of different elements with the same mass number.
- **Whole-number ratios don't always hold:** Complex organic compounds (e.g. sugars) can show more complicated ratios.
- **Cannot explain allotropes:** Diamond and graphite are both pure carbon but behave very differently – Dalton's theory gives no reason why.
- **No explanation of chemical bonding:** The theory says atoms combine, but not *why* or *how* – it says nothing about forces holding atoms together.

Need for a New Atomic Model

Dalton's theory treated the atom as a solid, structure-less sphere – the smallest possible particle of matter. This idea held for nearly 90 years, until:

- **1897 – J.J. Thomson's** cathode ray tube experiments discovered the **electron**, a negatively charged particle present inside *every* atom.
- Since atoms are electrically neutral overall, this meant every atom must *also* contain enough **positive charge** to balance the electrons.

This directly contradicted Dalton's claim that the atom is **indivisible**. If atoms contain smaller, charged particles inside them, a new question arises:

*"How are the electrons and the positive charge actually arranged **inside** the atom?"*

Answering this question required a completely new model of atomic structure – which is exactly what **J.J. Thomson's own "plum-pudding" model** (our next topic) attempted to do.

Quick Revision Tip

Remember: Every limitation of Dalton's theory corresponds to the discovery of a subatomic particle or a new type of atom (isotope/isobar). Link each limitation to *what* was discovered and *when* – these pairings are a favourite for 1-mark and 2-mark questions.

THOMSON'S ATOMIC MODEL (1904)

Dalton's theory had no answer for what was *inside* an atom. That changed in 1897, when **J.J. Thomson** (Cambridge) discovered the **electron** using his cathode ray tube experiment – becoming the first person to identify a subatomic particle.

What Did Thomson Discover?

In a cathode ray tube (a high-voltage discharge tube at low pressure), Thomson observed a stream of rays travelling from the cathode to the anode. He found that these rays:

- Were deflected **towards the positive plate** of an external electric field ⇒ the rays are **negatively charged**.
- Had a fixed charge-to-mass ratio (e/m) **regardless of the gas** used in the tube or the metal used for the electrodes.

This proved these negatively charged particles – named **electrons** – exist inside **every** atom, of every element. Since atoms are electrically neutral, Thomson reasoned every atom must also contain enough **positive charge** to cancel out its electrons.

The Plum Pudding (Watermelon) Model

In 1904, Thomson proposed a model to show *how* electrons and positive charge are arranged inside the atom:

- The atom is a **sphere of positive charge**, spread **uniformly** throughout, of radius $\sim 10^{-10}$ m.
- **Electrons are embedded** inside this sphere, like seeds in a fruit, at fixed positions.
- The negative charge of the electrons exactly balances the positive charge ⇒ atom is electrically **neutral** overall.

Popular names for this model:

- **Plum pudding model** – electrons (plums) scattered in a positively charged pudding.
- **Watermelon model** – positive charge (red flesh) with electrons (black seeds) embedded in it.

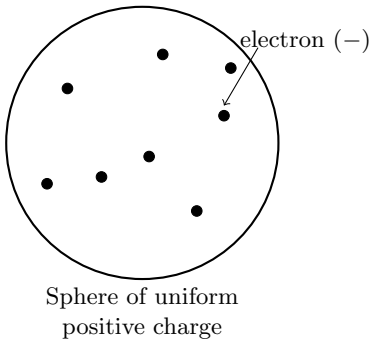


Figure 1: Thomson's plum pudding / watermelon model

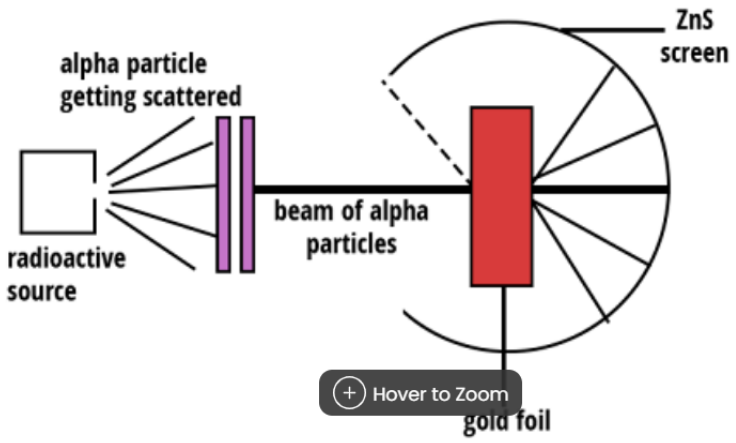
Limitations of Thomson's Model

- **No nucleus:** Assumed positive charge is spread out uniformly – it did not predict a small, dense, central nucleus.
- **Could not explain α-particle scattering:** When tested later by Rutherford, this model predicted *all* α-particles should pass through a thin foil with little to no deflection – but a small fraction bounced back sharply, which this model could not explain at all.
- **Could not explain atomic stability:** Gave no reason why the embedded electrons would stay in place rather than collapsing or rearranging.
- **Could not explain atomic spectra:** Gave no mechanism for why atoms emit light only at specific, sharp wavelengths (spectral lines).

Quick Revision Tip

Remember: Thomson's model got the *electron* right but the *arrangement* wrong. It was disproved by his own former student, **Ernest Rutherford**, through the α-particle scattering experiment – our next topic.

7. RUTHERFORD'S α-PARTICLE SCATTERING EXPERIMENT (1911)



To test Thomson's plum pudding model, **Ernest Rutherford** – along with his students **Hans Geiger** and **Ernest Marsden** – bombarded a thin gold foil with fast-moving, positively charged α-particles and studied how they scattered. Gold was chosen because it can be beaten into an extremely thin foil (about 2.1×10^{-7} m, only a few hundred atoms thick).

Experimental Setup

- A **radioactive source** (e.g. $^{214}_{84}$Po) emits a narrow, collimated **beam of α-particles** (energy $\sim$5.5 MeV) through a lead slit.
- The beam strikes a **thin gold foil**.
- A **rotatable detector** – a **ZnS (zinc sulphide) fluorescent screen** attached to a microscope – is placed all around the foil in a circular arrangement.
- Whenever an α-particle strikes the ZnS screen, it produces a tiny flash of light called a **scintillation**, which is counted at each angle θ to find how many particles scatter in that direction.

Observations

1. Most α-particles ($\sim$99.9%) passed straight through the foil with **little or no deflection**.
2. A small fraction were deflected through **moderate angles**.
3. A **very small fraction** (about 1 in 8000) were deflected through angles **greater than 90°**.
4. An extremely small number bounced back almost the way they came, i.e. deflected through nearly **180°**.

Conclusions Drawn

- Since most particles passed through undeflected, **most of the atom is empty space**.
- Since a few particles were deflected sharply, there must be a **small, dense, positively charged region** at the centre of the atom capable of repelling the positive α-particles strongly – this is the **nucleus**.
- Since very few particles bounced back at large angles, the nucleus occupies only a **tiny fraction** of the atom's total volume.
- Nearly the **entire mass** of the atom is concentrated in this nucleus.

Rutherford's Nuclear Model of the Atom

Based on these conclusions, Rutherford proposed a new model:

- The entire positive charge and almost all the mass of the atom are concentrated in a tiny central core – the **nucleus** (radius $\sim 10^{-14}$ to 10^{-15} m).
- **Electrons revolve around the nucleus** in circular orbits, much like planets around the Sun, at distances $\sim 10^4$ to 10^5 times the nuclear radius – so the atom (radius $\sim 10^{-10}$ m) is almost entirely empty space.
- The centripetal force needed for this circular motion is provided by the electrostatic force of attraction between the nucleus and the electrons.

This is why Rutherford's model is also called the **"planetary model"** or **"nuclear model"** of the atom.

Important Formulas – PUC Point of View

1. Distance of Closest Approach (r_0):

This is the minimum distance an α-particle gets to the nucleus in a **head-on collision** ($b = 0$). At this point, all of the α-particle's kinetic energy K has converted into electrostatic potential energy, and its velocity momentarily becomes zero.

$$K = \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(2e)(Ze)}{r_0} \quad \Longrightarrow \quad \boxed{r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{K}}$$

where Ze = charge of the nucleus, $2e$ = charge of the α-particle, K = initial kinetic energy of the α-particle.

2. Impact Parameter (b):

The impact parameter is the **perpendicular distance** between the initial velocity vector of the α-particle and the centre of the nucleus (i.e. how "off-centre" the particle's original path was). It determines the scattering angle θ:

$$\boxed{b = \frac{Ze^2}{4\pi\varepsilon_0\, K} \cot\left(\frac{\theta}{2}\right)}$$

Key relation: Smaller impact parameter $b \Rightarrow$ closer approach to the nucleus $\Rightarrow$ larger scattering angle θ.

Case	Result
Large b (far from nucleus)	$\theta \to 0^\circ$ (no deflection)
$b = 0$ (head-on collision)	$\theta = 180^\circ$ (bounces straight back)

Key takeaway: Rutherford himself called the large-angle scattering the most surprising result of his career – he compared it to firing a heavy shell at tissue paper and having it bounce straight back. **This experiment is one of the highest-yield topics in the chapter** – both formulas above, plus the observations-to-conclusions mapping, are frequently asked as 2-mark and 3-mark questions.

8. DISTANCE OF CLOSEST APPROACH (r_0)

Definition: Distance of Closest Approach

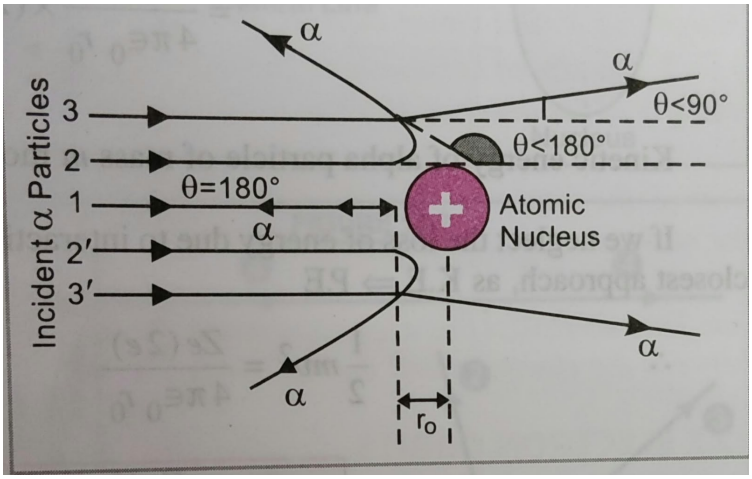
The **distance of closest approach** (r_0) is the minimum distance between the centre of the nucleus and an α-particle fired **head-on** at it, at which the particle's entire initial kinetic energy has been converted into electrostatic potential energy, and it momentarily comes to rest before being repelled back along its original path.

What is it?

Consider an α-particle fired **head-on** (i.e. directly towards the centre) at a nucleus. As it approaches, the positively charged nucleus **repels** it (Coulomb repulsion between like charges). This repulsion continuously slows the α-particle down, converting its kinetic energy into electrostatic potential energy.
At one particular point, the α-particle **momentarily comes to rest** – its entire kinetic energy has been converted into potential energy. It then gets pushed back the way it came (scattered through 180°).
The distance between the α-particle and the centre of the nucleus **at this instant** is called the **distance of closest approach**, denoted r_0.

Why is it Important?

- It gives us a way to estimate the **upper limit on the size of the nucleus** – since the α-particle never actually touches or enters the nucleus in this calculation, the true nuclear radius must be **less than** r_0.
- It was the **first experimental evidence** for how incredibly small and dense the nucleus is compared to the atom as a whole.
- It directly connects a **measurable quantity** (the α-particle's kinetic energy, known from the radioactive source used) to an otherwise **unmeasurable quantity** (the size of something 10^5 times smaller than the atom).
- It is a classic, high-weightage **numerical problem** topic in board and competitive exams.



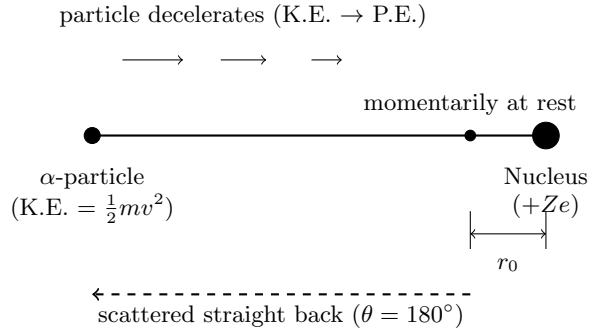


Figure 2: Head-on collision: α-particle decelerates, momentarily stops at distance r_0 from the nucleus, then is repelled straight back

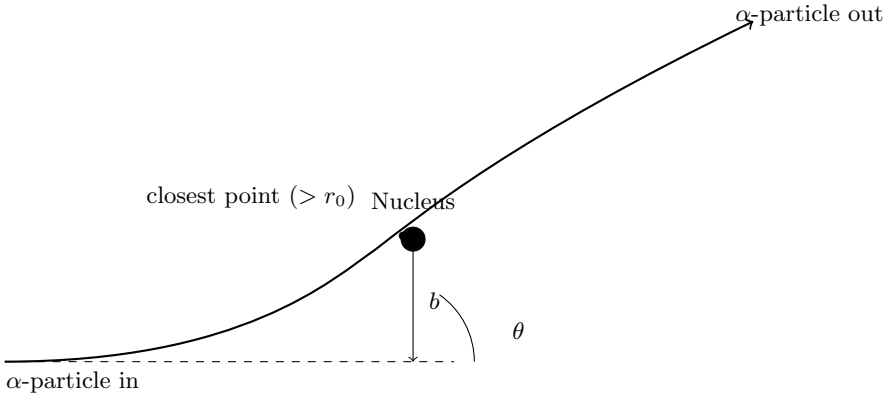


Figure 3: General case ($b \neq 0$): the α-particle follows a curved (hyperbolic) path and is deflected through angle θ, without ever reaching r_0 – only a head-on shot ($b = 0$) reaches the true distance of closest approach

Derivation of the Expression for r_0

Step 1 – Initial Kinetic Energy of the α-particle:
Far away from the nucleus, the α-particle (mass m, moving with speed v) has only kinetic energy and no potential energy (nucleus's electric field is negligible at large distance):

$$K = \frac{1}{2}mv^2$$

Step 2 – Electrostatic Potential Energy at Distance r_0:
The α-particle carries charge $+2e$ (2 protons), and the nucleus carries charge $+Ze$ (Z = atomic number). By Coulomb's law, the electrostatic potential energy of this pair of charges when separated by a distance r_0 is:

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(2e)(Ze)}{r_0} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{r_0}$$

Step 3 – Apply the Law of Conservation of Energy:
At the point of closest approach, the α-particle is momentarily at rest, so its kinetic energy is zero and **all** of the initial kinetic energy K has been converted into potential energy U:

$$K = U$$

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{r_0}$$

Step 4 – Solve for r_0:
Rearranging for r_0:

$$\boxed{r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{K} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{\frac{1}{2}mv^2}}$$

Where:

- Z = atomic number of the target nucleus (e.g. $Z = 79$ for gold)
- e = elementary charge $= 1.6 \times 10^{-19}$ C
- $K = \frac{1}{2}mv^2$ = initial kinetic energy of the α-particle
- ε_0 = permittivity of free space

Worked Example

For a gold nucleus ($Z = 79$) and an α-particle of kinetic energy 5.5 MeV ($= 5.5 \times 1.6 \times 10^{-13}$ J):

$$r_0 = \frac{2 \times 9 \times 10^9 \times 79 \times (1.6 \times 10^{-19})^2}{5.5 \times 1.6 \times 10^{-13}} \approx 4.1 \times 10^{-14} \text{ m}$$

This is many times larger than the actual nuclear radius ($\sim 10^{-15}$ m) – confirming r_0 only gives an **upper bound**, since the α-particle is repelled before it ever reaches the true edge of the nucleus.

Key takeaway: r_0 is calculated assuming a **head-on collision** ($b = 0$, $\theta = 180°$). Always start numericals on this topic by converting the given kinetic energy into **joules** before substituting – this is the single most common mistake students make.

IMPACT PARAMETER (b)

Definition

The **impact parameter (b)** is the **perpendicular distance** between the initial velocity vector of the α-particle and the centre of the nucleus, measured **when the α-particle is still far away** from the nucleus (before it feels any significant force from it).
In simple words: if the α-particle were to continue in a straight line (ignoring the nucleus's repulsion), b is how far off-centre that straight-line path would pass from the nucleus.

Why is Impact Parameter Needed?

- Not every α-particle in the beam is aimed exactly at the nucleus. Different particles pass the nucleus at different "off-centre" distances – b is what quantifies this.
- It **explains why the scattering angle θ varies** from particle to particle in the same experiment, even though every particle has the same initial energy.
- It connects a **geometric idea** (how close the particle's path was aimed to the nucleus) to a **measurable result** (the angle at which it eventually scatters) – this is the mathematical link that lets Rutherford's data be used to study the nucleus at all.
- Without this concept, distance of closest approach (r_0) alone only explains the special head-on case ($b = 0$) – impact parameter generalises the picture to **all** scattering angles observed in the experiment.

Expression for Impact Parameter

For scattering of an α-particle by the Coulomb (inverse-square) field of a nucleus, the impact parameter b is related to the scattering angle θ by:

$$\boxed{b = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2 \cot(\theta/2)}{K}}$$

where $K = \frac{1}{2}mv^2$ is the initial kinetic energy of the α-particle, Z = atomic number of the nucleus, and e = elementary charge.

(At PUC level this formula is used directly – its full derivation involves integrating the equation of motion under the Coulomb force and is beyond the syllabus.)

Two Important Limiting Cases

- b **large** (particle aimed far from nucleus) $\Rightarrow \cot(\theta/2)$ small $\Rightarrow \theta \to 0°$: **negligible deflection**, particle passes through almost undisturbed.
- $b = 0$ (particle aimed directly at nucleus, head-on) $\Rightarrow \cot(\theta/2) \to \infty \Rightarrow \theta = 180°$: particle is **scattered straight back**. This is exactly the head-on case used to derive r_0.

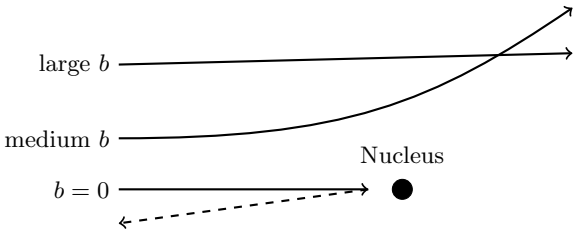


Figure 4: Smaller $b \Rightarrow$ closer approach $\Rightarrow$ larger scattering angle θ

Key takeaway: Smaller impact parameter $\Rightarrow$ larger scattering angle – this single inverse relationship explains *every* observation of the gold foil experiment: most particles (large b) pass straight through, while the rare few aimed almost directly at a nucleus (small b) scatter at large angles.

RUTHERFORD'S ATOMIC MODEL (1911)

Rutherford proposed the **nuclear model of the atom** based on the observations of the α-particle scattering experiment. The model replaced Thomson's "plum-pudding" model and introduced the concept of a small, dense, positively charged **nucleus** at the centre of the atom.

Rutherford's Nuclear Model

According to Rutherford's model, an atom consists of a tiny, dense, positively charged nucleus at its centre, with electrons revolving around the nucleus. Most of the volume of the atom is empty space.

Postulates of Rutherford's Atomic Model

1. **Presence of a central nucleus:**
   Almost the entire positive charge of the atom is concentrated in a very small region at the centre called the **nucleus**.
2. **Nucleus contains most of the mass:**
   Almost the entire mass of the atom is concentrated in the nucleus. Hence, the nucleus is extremely dense compared with the rest of the atom.
3. **Nucleus is extremely small:**
   The size of the nucleus is much smaller than the size of the atom. The typical atomic size is of the order of 10^{-10} m, whereas the nuclear size is of the order of 10^{-15} m.
4. **Electrons revolve around the nucleus:**
   Electrons are present outside the nucleus and revolve around it in circular orbits, somewhat similar to planets revolving around the Sun. Hence, Rutherford's model is also called the **planetary model** of the atom.
5. **Electrostatic force provides centripetal force:**
   The electrostatic force of attraction between the positively charged nucleus and the negatively charged electron provides the necessary centripetal force for the circular motion of the electron.
6. **Atom is electrically neutral:**
   The total positive charge of the nucleus is equal in magnitude to the total negative charge of the electrons. Therefore, an atom as a whole is electrically neutral.
7. **Most of the atom is empty space:**
   Since the nucleus occupies an extremely small fraction of the total volume of the atom, most of the atom is **empty space**.

Structure of the Atom

- **Centre:** tiny nucleus
- **Nucleus:** positively charged
- **Mass:** almost entirely concentrated in nucleus
- **Electrons:** revolve around nucleus
- **Space:** most of the atom is empty
- **Force:** electrostatic attraction

Important Orders of Magnitude

Quantity	Order of magnitude
Atomic radius	$\sim 10^{-10}$ m
Nuclear radius	$\sim 10^{-15}$ m
Atom : nucleus size	$\sim 10^{5} : 1$

Why is Rutherford's Model called the "Planetary Model"?

Rutherford compared the structure of an atom with the solar system:

Atom	Solar System	Analogy
Nucleus	Sun	Central body
Electron	Planet	Revolving body
Electrostatic force	Gravitational force	Attractive force

Thus, the model is commonly called the **planetary model of the atom**.

Important Observation

The Rutherford model successfully explained the results of the α-particle scattering experiment, particularly the observation that most α-particles passed through the foil without significant deflection while a very small fraction underwent large-angle scattering.
This led to the conclusion that the atom is mostly empty space and that its positive charge and most of its mass are concentrated in a tiny nucleus.

Major Limitation of Rutherford's Model

According to classical electromagnetic theory, a charged particle moving in a circular orbit is an **accelerating charge**. An accelerating charge should continuously radiate electromagnetic energy.
Therefore, an electron revolving around the nucleus should continuously lose energy. Its orbit would gradually shrink and the electron should eventually spiral into the nucleus.
Hence, according to classical physics:

Rutherford's atom should be unstable

But atoms are actually stable.

Therefore, Rutherford's model could not explain the stability of the atom.

KEY TAKEAWAY:
Rutherford ⇒ Nucleus + Empty Space + Revolving Electrons

The model explained the **nuclear structure** of the atom, but it could not explain **atomic stability** or the **line spectra of atoms**.

ELECTRON ORBITS

Rutherford's Atomic Model

What is an Electron Orbit?

According to Rutherford's atomic model, the electron revolves around the positively charged nucleus in a circular orbit. For a hydrogen atom:

- Charge of proton (nucleus) $= +e$
- Charge of electron $= -e$
- Mass of electron $= m$
- Speed of electron $= v$
- Radius of orbit $= r$

Since the electron moves in a circular path, its velocity continuously changes in direction. Therefore, the electron has **centripetal acceleration** directed towards the nucleus.

What keeps the electron in its orbit?

Two ideas are involved:

1. To keep an object moving in a circular path, a **centripetal force** is required.
2. The electron and nucleus are oppositely charged. Therefore, they attract each other through the **electrostatic force**.

Hence, in Rutherford's model, the electrostatic attraction between the nucleus and electron provides the required centripetal force.

$$\boxed{F_{\mathrm{e}} = F_{\mathrm{c}}}$$

Derivation of Electron Orbit

For an electron of mass m, charge $-e$, moving with speed v in a circular orbit of radius r, the electrostatic force provides the centripetal force.

$$F_{\mathrm{e}} = F_{\mathrm{c}} \quad \Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r}$$

$$\Rightarrow \quad mv^2 = \frac{e^2}{4\pi\varepsilon_0 r} \quad \Rightarrow \quad \boxed{v^2 = \frac{e^2}{4\pi\varepsilon_0 mr}} \quad \Rightarrow \quad \boxed{v = \sqrt{\frac{e^2}{4\pi\varepsilon_0 mr}}}$$

Also,

$$\boxed{r = \frac{e^2}{4\pi\varepsilon_0 mv^2}}$$

What does this equation tell us?

From

$$r = \frac{e^2}{4\pi\varepsilon_0 m v^2},$$

we obtain

$$\boxed{r \propto \frac{1}{v^2}}$$

Therefore:

Electron speed	Orbit radius
Increases	Decreases
Decreases	Increases

Thus, a faster electron corresponds to a smaller circular orbit in the classical Rutherford picture.

Kinetic Energy of the Electron

The kinetic energy of the revolving electron is

$$K = \frac{1}{2}mv^2 \quad \Rightarrow \quad K = \frac{1}{2}\left(\frac{e^2}{4\pi\varepsilon_0 r}\right) \quad \Rightarrow \quad \boxed{K = \frac{e^2}{8\pi\varepsilon_0 r}}$$

Potential Energy of the Electron

The electrostatic potential energy between two charges is

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

For the proton and electron, $q_1 = +e$ and $q_2 = -e$:

$$U = \frac{1}{4\pi\varepsilon_0}\frac{(+e)(-e)}{r} \quad \Rightarrow \quad \boxed{U = -\frac{e^2}{4\pi\varepsilon_0 r}}$$

The negative sign indicates that the electron is **bound to the positively charged nucleus**.

Total Energy of the Electron

The total mechanical energy is

$$E = K + U = \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \quad E = \frac{e^2 - 2e^2}{8\pi\varepsilon_0 r} = \boxed{-\frac{e^2}{8\pi\varepsilon_0 r}}$$

Therefore,

$$\boxed{E = -K} \qquad \text{and} \qquad \boxed{U = -2K}$$

Important Energy Relations

For an electron in a circular orbit of Rutherford's model:

1.

$$\boxed{K = \frac{e^2}{8\pi\varepsilon_0 r}}$$

2.

$$\boxed{U = -\frac{e^2}{4\pi\varepsilon_0 r}}$$

3.

$$\boxed{E = -\frac{e^2}{8\pi\varepsilon_0 r}}$$

4.

$$\boxed{U = -2K}$$

5.

$$\boxed{E = K + U = -K}$$

6.

$$\boxed{|E| = K}$$

Physical Meaning of Negative Total Energy

The total energy of the electron is negative:

$$E < 0$$

This means that the electron is **bound to the nucleus**.
To remove the electron completely from the atom and take it to infinity, energy must be supplied.
At infinity,

$$E = 0$$

Therefore, the energy required to remove the electron is equal to

$$\boxed{|E|}$$

for that particular orbit.

ATOMIC SPECTRA

What is an Atomic Spectrum?

When the radiation emitted or absorbed by atoms is analysed according to its wavelength or frequency, the resulting pattern is called an **atomic spectrum**.
Unlike a continuous spectrum, an atomic spectrum generally consists of radiation at specific wavelengths or frequencies.

Atomic spectrum = characteristic spectrum of an atom

Each element has a characteristic atomic spectrum, which can be used to identify the element.

Why do Atoms Produce Spectral Lines?

Electrons in an atom can occupy only certain allowed energy states. When an electron moves from a higher energy state E_i to a lower energy state E_f, a photon is emitted with energy

$$h\nu = E_i - E_f$$

Since the allowed energy levels are discrete, the possible energy differences are also discrete. Hence, only certain frequencies and wavelengths are emitted.

Discrete energy levels $\Rightarrow$ Discrete energy differences $\Rightarrow$ Discrete spectral lines

Emission Spectrum

The spectrum obtained when the radiation emitted by an excited substance is analysed is called an **emission spectrum**.
When electrons fall from higher to lower energy levels, photons are emitted.

$$E_i > E_f \qquad \Rightarrow \qquad h\nu = E_i - E_f$$

An atomic emission spectrum consists of characteristic **bright lines**.

Emission spectrum $\rightarrow$ Bright lines

Absorption Spectrum

The spectrum obtained when radiation passes through a substance and certain wavelengths are absorbed is called an **absorption spectrum**.
Atoms absorb photons whose energies match the energy difference between allowed energy states.
An absorption spectrum consists of characteristic **dark lines** superimposed on a continuous spectrum.

Absorption spectrum $\rightarrow$ Dark lines

Continuous Spectrum and Line Spectrum

Type	Characteristic	Example
Continuous spectrum	Contains a continuous range of wavelengths	White light
Line spectrum	Contains only specific wavelengths	Hydrogen atomic spectrum

Hydrogen Atomic Spectrum

Hydrogen is the simplest atom, containing one electron and one proton. When hydrogen atoms are excited, the electron can make transitions between different energy levels. Each transition is associated with a definite energy difference and hence a definite photon frequency.

$$\boxed{E_i - E_f = h\nu = \frac{hc}{\lambda}}$$

Therefore, hydrogen produces a characteristic **line spectrum**.
The spectral lines of hydrogen can be grouped into different families called **spectral series**.

Hydrogen spectrum → Spectral lines → Spectral series

What is a Spectral Series?

A **spectral series** is a group of spectral lines produced by transitions in which the electron falls from different higher energy levels to the **same lower energy level**.
For example, all transitions terminating at $n_f = 2$ form the **Balmer series**.

$$n_i = 3, 4, 5, \ldots \quad \longrightarrow \quad n_f = 2$$

Similarly,

Series	Final energy level
Lyman	$n_f = 1$
Balmer	$n_f = 2$
Paschen	$n_f = 3$
Brackett	$n_f = 4$
Pfund	$n_f = 5$
Humphreys	$n_f = 6$

KEY IDEA

Energy levels → Transitions → Photons → Spectral lines → Spectral series

SPECTRAL SERIES OF HYDROGEN

Hydrogen Spectrum

When hydrogen gas is excited electrically, the emitted radiation, when analysed using a spectroscope, does not form a continuous spectrum. Instead, it consists of a series of sharp lines at definite wavelengths.
These characteristic lines constitute the **hydrogen emission spectrum**.

Hydrogen spectrum = discrete spectral lines

In **1885**, the Swiss mathematician and teacher **Johann Jakob Balmer (1825–1898)** studied the measured wavelengths of the visible spectral lines of hydrogen.
He discovered that these wavelengths could be represented with remarkable accuracy by a simple mathematical relation.
Balmer's work was an **empirical** result: he found a mathematical pattern in the experimental data without providing a physical theory for the origin of the spectral lines.

Balmer: empirical description, not physical explanation

Meaning of an Empirical Relation

An **empirical relation** is a mathematical relationship obtained from experimental observations.
In simple terms:

Experimental data $\longrightarrow$ Observed pattern $\longrightarrow$ Mathematical formula

Balmer followed this approach to describe the wavelengths of hydrogen spectral lines.

Visible Lines of Hydrogen

The prominent visible hydrogen lines are approximately:

Line	Wavelength	Region / Colour
H_α	656.3 nm	Red
H_β	486.1 nm	Blue-green
H_γ	434.0 nm	Violet
H_δ	410.2 nm	Violet

These lines form the most prominent visible part of the **Balmer series**.

Balmer Series

The set of hydrogen spectral lines described by Balmer's formula is called the **Balmer series**.
In the later quantum interpretation, these lines correspond to electron transitions terminating at the second energy level:

$$n_i = 3, 4, 5, 6, \ldots \quad \longrightarrow \quad n_f = 2$$

Thus,

$$\begin{aligned} 3 \to 2 &\quad \Rightarrow \quad H_\alpha \\ 4 \to 2 &\quad \Rightarrow \quad H_\beta \\ 5 \to 2 &\quad \Rightarrow \quad H_\gamma \\ 6 \to 2 &\quad \Rightarrow \quad H_\delta \end{aligned}$$

Balmer Formula in Rydberg Form

Taking the reciprocal of Balmer's equation and introducing the Rydberg constant gives

$$\boxed{\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right)}$$

where

$$\boxed{R_H \approx 1.097 \times 10^7 \, \mathrm{m}^{-1}}$$

and

$$\boxed{n = 3, 4, 5, \ldots}$$

This form makes Balmer's relation a special case of the more general **Rydberg formula**.

General Rydberg Formula

The wavelengths of the hydrogen spectral series are described by

$$\boxed{\frac{1}{\lambda} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)}$$

where

$$\boxed{n_i > n_f}$$

Here,

Quantity	Meaning
n_i	Initial (higher) energy level
n_f	Final (lower) energy level
R_H	Rydberg constant for hydrogen
λ	Wavelength of emitted radiation

THE STORY OF HYDROGEN SPECTRUM

Experimental observations → Balmer (1885) → Empirical formula → Rydberg → Bohr (1913)

Balmer described the mathematical pattern. Rydberg generalised it. Bohr later provided a physical explanation using quantised energy levels.

Spectral Series of Hydrogen

When an excited hydrogen atom emits radiation, the electron may transition from a higher energy level n_i to a lower energy level n_f.
All spectral lines corresponding to transitions ending at the same final energy level n_f constitute a **spectral series**.
The wavelengths are given by the Rydberg formula:

$$\boxed{\frac{1}{\lambda} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)} \qquad n_i > n_f$$

where

$$R_H \approx 1.097 \times 10^7 \, \mathrm{m}^{-1}.$$

1. Lyman Series

For the Lyman series,

$$\boxed{n_f = 1}$$

Therefore,

$$n_i = 2, 3, 4, 5, \ldots \to 1$$

and the Rydberg formula becomes

$$\boxed{\frac{1}{\lambda} = R_H \left(1 - \frac{1}{n_i^2}\right)}$$

where

$$\boxed{n_i = 2, 3, 4, \ldots}$$

The Lyman series lies in the **ultraviolet region**.

2. Balmer Series

For the Balmer series,

$$\boxed{n_f = 2}$$

Therefore,

$$n_i = 3, 4, 5, 6, \ldots \to 2$$

and

$$\boxed{\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)}$$

or

$$\boxed{\frac{1}{\lambda} = R_H \left(\frac{1}{4} - \frac{1}{n_i^2}\right)}$$

where

$$\boxed{n_i = 3, 4, 5, \ldots}$$

The Balmer series lies mainly in the **visible region**.
Important lines are

$$3 \to 2 = H_\alpha, \qquad 4 \to 2 = H_\beta,$$

$$5 \to 2 = H_\gamma, \qquad 6 \to 2 = H_\delta.$$

3. Paschen Series

For the Paschen series,

$$\boxed{n_f = 3}$$

Therefore,

$$n_i = 4, 5, 6, 7, \ldots \rightarrow 3$$

and

$$\boxed{\frac{1}{\lambda} = R_H \left(\frac{1}{3^2} - \frac{1}{n_i^2} \right)}$$

or

$$\boxed{\frac{1}{\lambda} = R_H \left(\frac{1}{9} - \frac{1}{n_i^2} \right)}$$

where

$$\boxed{n_i = 4, 5, 6, \ldots}$$

The Paschen series lies in the **infrared region**.

4. Brackett Series

For the Brackett series,

$$\boxed{n_f = 4}$$

Therefore,

$$n_i = 5, 6, 7, 8, \ldots \rightarrow 4$$

and

$$\boxed{\frac{1}{\lambda} = R_H \left(\frac{1}{4^2} - \frac{1}{n_i^2} \right)}$$

or

$$\boxed{\frac{1}{\lambda} = R_H \left(\frac{1}{16} - \frac{1}{n_i^2} \right)}$$

where

$$\boxed{n_i = 5, 6, 7, \ldots}$$

The Brackett series lies in the **infrared region**.

5. Pfund Series

For the Pfund series,

$$\boxed{n_f = 5}$$

Therefore,

$$n_i = 6, 7, 8, 9, \ldots \to 5$$

and

$$\boxed{\frac{1}{\lambda} = R_H \left(\frac{1}{5^2} - \frac{1}{n_i^2} \right)}$$

or

$$\boxed{\frac{1}{\lambda} = R_H \left(\frac{1}{25} - \frac{1}{n_i^2} \right)}$$

where

$$\boxed{n_i = 6, 7, 8, \ldots}$$

The Pfund series lies in the **infrared region**.

Hydrogen Spectral Series – Summary

Series	n_f	Transitions	Region
Lyman	1	$2, 3, 4, \ldots \to 1$	Ultraviolet
Balmer	2	$3, 4, 5, \ldots \to 2$	Visible
Paschen	3	$4, 5, 6, \ldots \to 3$	Infrared
Brackett	4	$5, 6, 7, \ldots \to 4$	Infrared
Pfund	5	$6, 7, 8, \ldots \to 5$	Infrared

$$\boxed{\text{Lyman} \to 1, \quad \text{Balmer} \to 2, \quad \text{Paschen} \to 3, \quad \text{Brackett} \to 4, \quad \text{Pfund} \to 5}$$

One Formula for All Series

Students need not memorise five independent equations.
The single Rydberg equation is

$$\boxed{\frac{1}{\lambda} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)}$$

Simply substitute the value of n_f corresponding to the required series.

Series	Lyman	Balmer	Paschen	Brackett	Pfund
n_f	1	2	3	4	5

BOHR MODEL OF HYDROGEN ATOM

What is the Bohr Model?

The **Bohr model of the hydrogen atom**, proposed by **Niels Bohr in 1913**, describes the electron as revolving around the positively charged nucleus only in certain permitted orbits.
These permitted orbits are called **stationary orbits** or **stationary states**.
Each stationary state has a definite energy.

$$\boxed{\text{Bohr model} = \text{discrete allowed orbits} + \text{discrete energies}}$$

Why Was the Bohr Model Needed?

Rutherford's nuclear model successfully explained the existence of a small, massive, positively charged nucleus, but it had two major difficulties.

1. **Atomic stability:** According to classical electromagnetic theory, an accelerating electron should continuously radiate energy. Therefore, the electron should lose energy and spiral into the nucleus.

   $$\text{Accelerating charge} \Rightarrow \text{radiation} \Rightarrow \text{energy loss} \Rightarrow \text{collapse of atom}$$

   This contradicts the observed stability of atoms.

2. **Line spectra:** Rutherford's classical model could not explain why hydrogen emits only specific wavelengths instead of a continuous range of wavelengths.

Hence, a new model incorporating the emerging quantum ideas was required.

$$\boxed{\text{Bohr model} \Rightarrow \text{stable atom} + \text{discrete energy levels}}$$

Central Idea of Bohr

The electron cannot occupy an arbitrary orbit around the nucleus. Only certain **allowed stationary orbits** are possible.
Each allowed orbit has a definite energy.
Therefore,

$$\boxed{\text{Allowed orbits} \Rightarrow \text{Allowed energies}}$$

and transitions between these allowed energies produce or absorb photons.

$$\boxed{\text{Energy levels} \rightarrow \text{Electron transition} \rightarrow \text{Photon} \rightarrow \text{Spectral line}}$$

Bohr's First Postulate – Stationary Orbits

The electron can revolve around the nucleus only in certain permitted orbits called **stationary orbits** or **stationary states**.
While the electron remains in a stationary orbit, it does **not emit electromagnetic radiation**.
Each stationary state has a definite total energy.

$$\boxed{\text{Stationary orbit} \Rightarrow \text{No continuous radiation}}$$

Thus, an electron can remain in an allowed orbit without continuously losing energy.

Bohr's Second Postulate – Quantisation of Angular Momentum

Only those orbits are allowed for which the angular momentum of the electron is an integral multiple of $\frac{h}{2\pi}$.
Thus,

$$L = mvr = \frac{nh}{2\pi}, \qquad n = 1, 2, 3, \ldots$$

Hence,

$$\boxed{mvr = \frac{nh}{2\pi} = n\hbar}$$

where

$$\boxed{\hbar = \frac{h}{2\pi}}$$

The integer n is called the **principal quantum number**.
Therefore, the angular momentum is **quantised**.

$$\boxed{L = \hbar,\ 2\hbar,\ 3\hbar, \ldots}$$

Bohr's Third Postulate – Radiation During Transition

An electron can make a transition from one stationary state to another.
When an electron moves from a higher energy state E_i to a lower energy state E_f, a photon is emitted.
The photon energy is equal to the difference between the two energies:

$$\boxed{h\nu = E_i - E_f} \qquad (E_i > E_f)$$

Since

$$E_\gamma = h\nu = \frac{hc}{\lambda},$$

we also have

$$\boxed{\frac{hc}{\lambda} = E_i - E_f}$$

Thus, every permitted transition corresponds to a definite photon frequency and hence to a definite spectral line.

Absorption of Radiation

If an electron absorbs a photon of exactly the required energy, it can move from a lower energy state to a higher energy state.
Therefore,

$$\boxed{E_f - E_i = h\nu} \qquad (E_f > E_i)$$

Hence,

$$\boxed{\begin{array}{lcl} \text{Emission} & : & h\nu = E_i - E_f \\ \text{Absorption} & : & h\nu = E_f - E_i \end{array}}$$

Meaning of the Principal Quantum Number

The allowed stationary states are labelled by

$$\boxed{n = 1, 2, 3, 4, \ldots}$$

The state $n = 1$ is called the **ground state**.
The states

$$n = 2, 3, 4, \ldots$$

are called **excited states**.
Thus,

$$\boxed{n = 1 \Rightarrow \text{Ground state}}$$

$$\boxed{n > 1 \Rightarrow \text{Excited state}}$$

THE CENTRAL IDEA OF THE BOHR MODEL

Only certain orbits are allowed
$\Downarrow$
Only certain energies are allowed
$\Downarrow$
Electron jumps between allowed levels
$\Downarrow$
$h\nu = E_i - E_f$
$\Downarrow$
Spectral line is produced

$$\boxed{\text{Discrete energy levels} \Rightarrow \text{Discrete energy differences} \Rightarrow \text{Discrete spectral lines}}$$

Bohr's Three Major Postulates – Quick Revision

1.	**Stationary orbits**	Electrons move only in certain permitted orbits and do not radiate energy while remaining in them.
2.	**Angular momentum quantisation**	$mvr = \dfrac{nh}{2\pi}, \qquad n = 1, 2, 3, \ldots$
3.	**Radiation during transition**	$h\nu = E_i - E_f$ for emission from a higher to a lower energy state.

Basic Equations of Bohr's Model

For a hydrogen atom, the electrostatic force provides the centripetal force:

$$\boxed{\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}}$$

and Bohr's quantisation condition is

$$\boxed{mvr = \frac{nh}{2\pi}} \qquad n = 1, 2, 3, \ldots$$

These two equations are used in all the following derivations.

1. Radius of Bohr's Stationary Orbit

From

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r} \Rightarrow mv^2 = \frac{e^2}{4\pi\varepsilon_0 r}$$

and

$$mvr = \frac{nh}{2\pi} \Rightarrow v = \frac{nh}{2\pi mr},$$

we get

$$m\left(\frac{nh}{2\pi mr}\right)^2 = \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \quad \boxed{r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2}}$$

Defining the Bohr radius,

$$\boxed{a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2} = 5.29 \times 10^{-11}\,\text{m}}$$

therefore,

$$\boxed{r_n = a_0 n^2}$$

Hence,

$$\boxed{r_n \propto n^2}$$

For hydrogen-like species:

$$\boxed{r_n = \frac{a_0 n^2}{Z}}$$

2. Velocity of Electron in Bohr's Stationary Orbit

From Bohr's quantisation condition,

$$mvr = \frac{nh}{2\pi} \Rightarrow v = \frac{nh}{2\pi mr}.$$

Using

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2},$$

we obtain

$$v_n = \frac{nh}{2\pi m}\left(\frac{\pi m e^2}{\varepsilon_0 n^2 h^2}\right)$$

$$\Rightarrow \quad \boxed{v_n = \frac{e^2}{2\varepsilon_0 hn}}$$

Hence,

$$\boxed{v_n \propto \frac{1}{n}}$$

Numerically,

$$\boxed{v_n = \frac{2.18 \times 10^6}{n}\,\mathrm{m\,s^{-1}}}$$

For hydrogen-like species,

$$\boxed{v_n = \frac{2.18 \times 10^6 Z}{n}\,\mathrm{m\,s^{-1}}}$$

3. Orbital Frequency of Electron

The frequency of revolution of the electron is

$$\nu_n = \frac{\text{distance in one revolution}}{\text{time for one revolution}} = \frac{2\pi r_n}{2\pi r_n / v_n}$$

$$\Rightarrow \quad \nu_n = \frac{v_n}{2\pi r_n}$$

Using

$$v_n = \frac{e^2}{2\varepsilon_0 hn} \qquad \text{and} \qquad r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2},$$

we get

$$\nu_n = \frac{1}{2\pi}\frac{e^2}{2\varepsilon_0 hn}\frac{\pi m e^2}{\varepsilon_0 n^2 h^2}$$

$$\Rightarrow \quad \boxed{\nu_n = \frac{me^4}{4\varepsilon_0^2 h^3 n^3}}$$

Hence,

$$\boxed{\nu_n \propto \frac{1}{n^3}}$$

Note: This is the orbital frequency of the electron, not the frequency of emitted radiation during an electronic transition.

4. Total Energy of Electron in Bohr's Stationary Orbit

The total energy is

$$E = K + U.$$

From the force equation,

$$mv^2 = \frac{e^2}{4\pi\varepsilon_0 r} \Rightarrow K = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\varepsilon_0 r}$$

and the electrostatic potential energy is

$$U = -\frac{e^2}{4\pi\varepsilon_0 r}.$$

Therefore,

$$E = \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \quad \boxed{E = -\frac{e^2}{8\pi\varepsilon_0 r}}$$

Using

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2},$$

we obtain

$$E_n = -\frac{e^2}{8\pi\varepsilon_0}\frac{\pi m e^2}{\varepsilon_0 n^2 h^2}$$

$$\Rightarrow \quad \boxed{E_n = -\frac{me^4}{8\varepsilon_0^2 h^2 n^2}}$$

Numerically,

$$\boxed{E_n = -\frac{2.18 \times 10^{-18}}{n^2}\,\text{J} = -\frac{13.6}{n^2}\,\text{eV}}$$

Hence,

$$\boxed{E_n \propto -\frac{1}{n^2}}$$

The negative sign indicates that the electron is bound to the nucleus.

Important Results of Bohr's Model

1.	$\boxed{r_n = a_0 n^2}$	$\boxed{r_n \propto n^2}$
2.	$\boxed{v_n = \frac{2.18 \times 10^6}{n}\,\text{m s}^{-1}}$	$\boxed{v_n \propto \frac{1}{n}}$
3.	$\boxed{\nu_n = \frac{me^4}{4\varepsilon_0^2 h^3 n^3}}$	$\boxed{\nu_n \propto \frac{1}{n^3}}$
4.	$\boxed{E_n = -\frac{13.6}{n^2}\,\text{eV}}$	$\boxed{E_n \propto -\frac{1}{n^2}}$

Bohr's Second Postulate and de Broglie's Explanation

Recap: Bohr's Second Postulate

The angular momentum of an electron revolving around the nucleus is quantised, i.e. it can only be an integral multiple of $\frac{h}{2\pi}$:

$$L_n = mv_n r_n = \frac{nh}{2\pi}, \qquad n = 1, 2, 3, \ldots$$

Bohr gave no physical reasoning for this condition — it was simply postulated to match experimental results.

The Puzzle: Why should angular momentum take *only* these particular quantised values? This question remained unanswered for **10 years**, until in **1923**, the French physicist **Louis de Broglie** provided a physical explanation using the wave nature of matter (his hypothesis proposed in 1924, formally verified by Davisson–Germer experiment in 1927).

de Broglie's Hypothesis (Recap from Ch. 11)

Every moving particle, including an electron, has a wave associated with it, called the **matter wave**, with wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

de Broglie's Explanation

de Broglie proposed that an electron in a Bohr orbit should be regarded as a **particle wave**, moving in a circle around the nucleus. For this wave to exist as a **stable, stationary pattern** (i.e. not destroy itself by destructive interference on successive revolutions), the circumference of the orbit **must contain a whole number of de Broglie wavelengths** – exactly like a **standing wave** on a stretched string whose ends are joined into a loop.

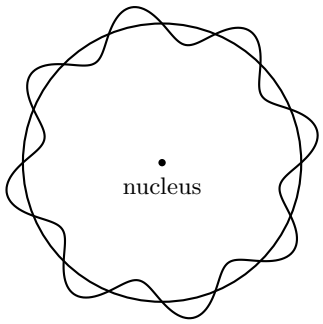


Figure 5: Standing wave pattern on a circular orbit (schematic, integral number of wavelengths fit the circumference)

Condition for a stable orbit:

$$\text{Circumference} = n \times (\text{de Broglie wavelength})$$

$$2\pi r_n = n\lambda, \qquad n = 1, 2, 3, \ldots$$

This is the resonance / standing-wave condition that permits only certain discrete orbits to exist.

Derivation: Recovering Bohr's Quantisation Condition

Step-by-step derivation

Step 1: Standing wave condition:

$$2\pi r_n = n\lambda \qquad \cdots (1)$$

Step 2: de Broglie wavelength of the electron:

$$\lambda = \frac{h}{mv_n} \qquad \cdots (2)$$

Step 3: Substituting (2) into (1):

$$2\pi r_n = n\left(\frac{h}{mv_n}\right)$$

Step 4: Rearranging:

$$mv_n r_n = \frac{nh}{2\pi}$$

Step 5: Since $L_n = mv_n r_n$ is the angular momentum,

$$\boxed{L_n = \frac{nh}{2\pi}}$$

This is **exactly Bohr's second postulate** – but now derived from a physical picture (wave nature of the electron) rather than simply assumed.

Key Takeaway

Bohr's quantisation of angular momentum is **not an arbitrary rule** – it is a natural consequence of treating the electron as a **de Broglie matter wave** that must form a **standing wave** around the nucleus. Only orbits satisfying $2\pi r_n = n\lambda$ are stable; all other orbits destructively interfere and cannot persist.

Quick Comparison

Aspect	**Bohr (1913)**	**de Broglie (1923)**
Nature of electron	Treated as a particle	Treated as a particle *and* wave
Reason for quantisation	Postulated (no explanation)	Explained using standing-wave condition
Condition	$L_n = \frac{nh}{2\pi}$	$2\pi r_n = n\lambda \Rightarrow$ same result

Exam Corner (2nd PUC)

- Frequently asked as a **3-mark derivation** question: "Derive Bohr's quantisation condition using de Broglie's hypothesis."
- Also asked as: "How does de Broglie's hypothesis explain the stability of Bohr orbits?"
- **Do not forget** to explicitly state the standing-wave / resonance condition – examiners specifically look for the phrase *"integral number of wavelengths fit the circumference"*.
- Diagram of the standing wave on a circular orbit fetches extra credit – always draw it.

Limitations of Bohr's Model

Bohr's model successfully explained the stability of the atom and the hydrogen spectrum, but it is built on a mix of classical and quantum ideas and breaks down in several situations. NCERT explicitly points out **two main limitations** right after introducing de Broglie's explanation – these are the most important for board exams – followed by additional limitations commonly asked in PUC/CBSE question banks.

NCERT's Two Core Limitations

Limitation 1: Restricted to Hydrogenic (Single-Electron) Atoms

The Bohr model is applicable only to **hydrogenic atoms** – atoms/ions with a nucleus of charge $+Ze$ and a **single electron** (e.g. H, He^{+}, Li^{2+}). It **cannot be extended** even to a simple two-electron atom like neutral helium.
Reason: In a multi-electron atom, each electron interacts not only with the positively charged nucleus but also with **every other electron**. Bohr's model has no way to account for this electron–electron interaction/repulsion.

Limitation 2: Cannot Predict Relative Intensities of Spectral Lines

Bohr's theory allows us to calculate only the **frequencies** (or wavelengths) of spectral lines emitted during electron transitions. It gives **no information** about **why some spectral lines are more intense (brighter) than others**. The relative intensity of a spectral line depends on **how probable** a particular transition is – a concept Bohr's theory does not address at all; it required quantum mechanics (transition probabilities) to explain.

Additional Limitations (Frequently Asked)

1. **Violates Heisenberg's Uncertainty Principle:** Bohr's model assumes an electron has a precisely known orbit, i.e. simultaneously well-defined position and momentum. This directly contradicts the uncertainty principle, $\Delta x\,\Delta p \geq \dfrac{h}{4\pi}$.
2. **Ignores wave nature of the electron:** Bohr treated the electron purely as a particle moving in a fixed circular path, without incorporating de Broglie's matter-wave concept (only patched in later by de Broglie himself).
3. **Cannot explain the Zeeman Effect:** The splitting of spectral lines when the atom is placed in an **external magnetic field** is not explained by Bohr's model.
4. **Cannot explain the Stark Effect:** The splitting of spectral lines when the atom is placed in an **external electric field** is also left unexplained.
5. **No explanation for fine structure:** High-resolution spectroscopy reveals that spectral lines (e.g. in the Balmer series) are actually closely spaced groups of lines (**fine structure**). Bohr's model predicts only single lines.
6. **Arbitrary restriction to circular orbits:** Bohr's model assumes only circular orbits, with no justification for excluding elliptical orbits (later addressed by Sommerfeld's extension).
7. **No theoretical basis for quantisation (before de Broglie):** The angular momentum quantisation condition $L_n = \dfrac{nh}{2\pi}$ was **postulated**, not derived from first principles – it had to be justified later using de Broglie's hypothesis.

Summary Table

Limitation	What Bohr's Model Fails To Explain
Multi-electron atoms	Cannot be applied beyond hydrogenic (single-electron) systems
Relative intensities	Cannot predict why some spectral lines are brighter than others
Uncertainty Principle	Assumes exact orbit (known position & momentum simultaneously)
Wave nature	Treats electron only as a particle, ignoring matter waves
Zeeman Effect	Splitting of lines in a magnetic field
Stark Effect	Splitting of lines in an electric field
Fine structure	Closely spaced sub-lines within spectral lines
Orbit shape	No justification for circular-only orbits (vs. elliptical)

Key Takeaway

Bohr's model is a **hybrid** of classical mechanics (orbital motion) and an ad-hoc quantum condition (angular momentum quantisation). It works remarkably well for **hydrogen and hydrogenic ions** but fails wherever multi-electron interactions, wave nature of matter, or external fields come into play. These gaps were eventually resolved by **quantum mechanics** (Schrödinger's wave equation).

Exam Corner (2nd PUC)

- Very common as a **2-mark question**: "State any two limitations of Bohr's atomic model." – Always lead with the **two NCERT limitations** (multi-electron atoms & relative intensities) as the safest, textbook-aligned answer.
- If asked for **more than two**, add Heisenberg's Uncertainty Principle and Zeeman/Stark effect next – these appear most often in board papers.
- Common mistake: writing only vague statements like "it has drawbacks" without naming the specific effect (Zeeman/Stark) – examiners deduct marks for lack of specificity.
- This topic often appears combined with "de Broglie's explanation" as a single 5-mark long-answer question – keep both ready together.

ATOMS – MASTER FORMULA SHEET

Class 12 Physics – 2nd PUC / CBSE

This sheet compiles **every formula** in the Atoms chapter, verified against NCERT, with variable-by-variable explanation, physical meaning, typical units, and the numerical question types each formula is used for. Formulas are grouped by topic in the order they appear in the chapter.

1. Rutherford's Alpha-Particle Scattering

1.1 Coulomb Repulsive Force between α-particle and Nucleus

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{(2e)(Ze)}{r^2}$$

- F = electrostatic repulsive force (N)
- $2e$ = charge of α-particle (it is a He nucleus: 2 protons)
- Ze = charge of the target nucleus, Z = atomic number
- r = distance between α-particle and nucleus at that instant (m)
- $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ N m^{2}C^{-2} (Coulomb's constant)

Used for: Finding force at a given separation; basis for deriving trajectory and scattering angle.

1.2 Distance of Closest Approach (r_0)

$$r_0 = \frac{1}{4\pi\epsilon_0} \cdot \frac{2Ze^2}{K}$$

- r_0 = distance of closest approach (m) – gives an **upper estimate of nuclear size**
- Z = atomic number of target nucleus (e.g. $Z = 79$ for gold)
- e = electronic charge $= 1.6 \times 10^{-19}$ C
- $K = \frac{1}{2}mv^2$ = initial kinetic energy of the α-particle (J or MeV), measured far from the nucleus

Physical idea: At r_0, the α-particle momentarily stops (head-on collision) – all KE converts to electrostatic PE.
Used for: Direct numericals – "find distance of closest approach given KE of α-particle (in MeV) and Z." Very frequently asked (2–3 marks).

1.3 Impact Parameter (b)

$$b = \frac{Ze^2 \cot(\theta/2)}{4\pi\epsilon_0 K}$$

- b = impact parameter (m) – perpendicular distance between the initial velocity direction of the α-particle and the centre of the nucleus
- θ = scattering angle (angle by which the particle deflects from its original path)
- K = kinetic energy of the incident α-particle

Key relation: b and θ are **inversely related** – smaller b (closer to head-on) gives **larger** θ; large b gives small/negligible deflection.
Used for: Numericals asking to calculate b for a given scattering angle, or explain why most particles pass through undeflected (large b).

1.4 Rutherford Scattering Formula (conceptual, rarely numerical at PUC level)

$$N(\theta) \propto \frac{1}{\sin^4(\theta/2)}$$

$N(\theta)$ = number of α-particles scattered at angle θ. Explains why very few particles (~ 1 in 8000) scatter at $\theta > 90°$. Mostly asked as a **conceptual/assertion-reason** question, not a calculation.

2. Bohr's Model – Orbit Quantities (Hydrogen & Hydrogenic Atoms)

2.1 Quantisation of Angular Momentum

$$L_n = mv_n r_n = \frac{nh}{2\pi}$$

- L_n = angular momentum of electron in n^{th} orbit (kg m^{2}s^{-1})
- m = mass of electron = 9.1×10^{-31} kg
- v_n = orbital speed of electron in n^{th} orbit
- r_n = radius of n^{th} orbit
- n = principal quantum number = $1, 2, 3, \ldots$
- h = Planck's constant = 6.63×10^{-34} J s

Used for: Foundation of Bohr model; combined with Coulomb force equation to derive r_n, v_n, E_n (see below). Also directly tested: "find angular momentum in n^{th} orbit."

2.2 Radius of n^{th} Orbit

$$r_n = \left(\frac{n^2 h^2 \epsilon_0}{\pi m Z e^2}\right) = 0.529 \times \frac{n^2}{Z} \text{ Å}$$

- r_n = radius of n^{th} Bohr orbit
- Z = atomic number ($Z = 1$ for hydrogen; use actual Z for hydrogenic ions like He^+, Li^{2+})
- n = orbit number (principal quantum number)
- 1 Å = 10^{-10} m
- For $n = 1, Z = 1$: $r_1 = 0.529$ Å – called the **Bohr radius** (a_0)

Key trend: $r_n \propto \frac{n^2}{Z}$ – radius increases rapidly with n, decreases with Z.
Used for: Finding orbit radius for given n and Z; comparing radii of different orbits/atoms; very common 1–2 mark numerical.

2.3 Speed of Electron in n^{th} Orbit

$$v_n = \frac{e^2}{2\epsilon_0 h} \cdot \frac{Z}{n} = 2.18 \times 10^6 \times \frac{Z}{n} \text{ m/s}$$

- v_n = orbital speed of electron in n^{th} orbit (m/s)
- 2.18×10^6 m/s = speed of electron in ground state of hydrogen ($n = 1, Z = 1$)

Key trend: $v_n \propto \frac{Z}{n}$ – speed **decreases** as n increases (unlike radius, which increases).
Used for: Numericals on electron speed at a given level; also used to show $v_n/c \approx 1/137 \times Z/n$ (fine structure constant context, occasionally asked).

2.4 Time Period and Frequency of Revolution

$$T_n = \frac{2\pi r_n}{v_n} \propto \frac{n^3}{Z^2} \qquad \nu_n = \frac{1}{T_n} \propto \frac{Z^2}{n^3}$$

- T_n = time period of revolution in n^{th} orbit (s) – time for one complete revolution
- ν_n = frequency of revolution (Hz) – number of revolutions per second (**not** to be confused with photon frequency during transitions)

Used for: "Find the frequency/time period of revolution of electron in n^{th} orbit." Derived using $r_n \propto n^2/Z$ and $v_n \propto Z/n$, so $T_n \propto n^3/Z^2$.

3. Energy of the Electron

3.1 Kinetic, Potential, and Total Energy

$$KE_n = \frac{1}{2}mv_n^2 = +\frac{1}{2}|U_n| \qquad PE_n = U_n = -\frac{Ze^2}{4\pi\epsilon_0 r_n}$$

$$E_n = KE_n + PE_n = -\frac{me^4Z^2}{8\epsilon_0^2 h^2 n^2} = -13.6\,\frac{Z^2}{n^2}\ \text{eV}$$

- E_n = total energy of electron in n^{th} orbit (**always negative** – electron is bound)
- -13.6 eV = ground state ($n = 1$) energy of hydrogen ($Z = 1$) – **most important constant in this chapter**
- $1\ \text{eV} = 1.6 \times 10^{-19}$ J

CRITICAL relation (very commonly tested):

$$PE_n = 2E_n \qquad \text{and} \qquad KE_n = -E_n = |E_n|$$

Used for: The single most numerically-tested formula in this chapter. Used to find energy at any level, energy released/absorbed during transitions, ionisation energy, excitation energy.

3.2 Energy Released/Absorbed During Transition

$$h\nu = E_2 - E_1 = 13.6\,Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\text{eV} \qquad (n_2 > n_1)$$

- $h\nu$ = energy of **photon** emitted (electron falls $n_2 \to n_1$) or absorbed (electron rises $n_1 \to n_2$)
- ν = frequency of the emitted/absorbed photon (Hz) – **different** from orbital revolution frequency above
- n_1, n_2 = initial and final orbit numbers (lower and higher energy levels)

Used for: Finding energy/frequency/wavelength of photon in a transition; the core numerical type of the whole chapter.

3.3 Ionisation Energy and Excitation Energy

$$\text{Ionisation Energy (from } n^{\text{th}} \text{ level)} = 0 - E_n = -E_n = 13.6\,\frac{Z^2}{n^2}\ \text{eV}$$

$$\text{Excitation Energy} = E_{\text{final}} - E_{\text{initial}}$$

- **Ionisation energy**: minimum energy to remove the electron completely from a given orbit to $n = \infty$ (where $E_\infty = 0$)
- **Ionisation energy of H (ground state)** = 13.6 eV – a must-remember constant
- **Excitation energy**: energy needed to move electron from ground state (or any lower state) to a specified higher state (electron stays bound)

Used for: "Find ionisation energy of H in n^{th} state" or "find first/second excitation energy" – extremely common question type.

4. Hydrogen Spectrum (Spectral Series)

4.1 Rydberg Formula (Wave Number)

$$\bar{\nu} = \frac{1}{\lambda} = R_H Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

- $\bar{\nu} = \dfrac{1}{\lambda}$ = wave number (number of waves per unit length, unit: m^{-1} or cm^{-1})
- λ = wavelength of emitted/absorbed spectral line
- R_H = Rydberg constant = $1.097 \times 10^7\ \mathrm{m}^{-1}$
- n_1 = lower orbit (series limit defines the series name); n_2 = higher orbit ($n_2 > n_1$)

Used for: The standard formula to compute wavelength of any spectral line – pair this with the series table below.

Series	n_1	**Region**	**Notes**
Lyman	1	Ultraviolet	$n_2 = 2, 3, 4, \ldots$; discovered 1906
Balmer	2	Visible	Only series with lines in visible region; $n_2 = 3$ gives H_α (first/longest-λ line)
Paschen	3	Infrared	$n_2 = 4, 5, \ldots$
Brackett	4	Infrared	$n_2 = 5, 6, \ldots$
Pfund	5	Infrared	$n_2 = 6, 7, \ldots$

4.2 Series Limit (Shortest Wavelength of a Series)

$$\frac{1}{\lambda_{\min}} = R_H Z^2 \left(\frac{1}{n_1^2} - \frac{1}{\infty^2} \right) = \frac{R_H Z^2}{n_1^2}$$

Occurs when $n_2 \to \infty$ (electron transitions from just outside the atom). **Longest wavelength** of a series occurs for the smallest jump, $n_2 = n_1 + 1$.
Used for: "Find the shortest/longest wavelength in the Lyman/Balmer series" – very common numerical.

4.3 Number of Spectral Lines

$$\text{Number of spectral lines} = \frac{n(n-1)}{2}$$

- n = the highest energy level the electron is initially excited to (all possible downward transitions from this level down to $n = 1$ are counted)

Used for: "An electron is excited to the n^{th} level; find the total number of spectral lines possible in the emission spectrum." Extremely common in board exams and MCQs.

5. de Broglie's Explanation (Connects Ch. 11 and Ch. 12)

5.1 Standing Wave / Quantisation Condition

$$2\pi r_n = n\lambda, \qquad \lambda = \frac{h}{mv_n} \quad \Longrightarrow \quad mv_n r_n = \frac{nh}{2\pi}$$

- λ = de Broglie wavelength of the orbiting electron
- Condition: circumference of orbit = integral number of de Broglie wavelengths

Used for: Derivation-type questions (already covered in the previous topic sheet); occasionally combined with numerical to find λ of electron in a given orbit.

6. Quick-Reference Constants Table

Constant	Value
Bohr radius ($a_0 = r_1$ for H)	0.529 Å$= 0.529 \times 10^{-10}$ m
Ground state energy of H	-13.6 eV
Ionisation energy of H	13.6 eV
Rydberg constant (R_H)	1.097×10^{7} m^{-1}
Speed of electron in ground state of H	2.18×10^{6} m/s
Electronic charge (e)	1.6×10^{-19} C
Planck's constant (h)	6.63×10^{-34} J s
Mass of electron (m)	9.1×10^{-31} kg
$\frac{1}{4\pi\epsilon_0}$	9×10^{9} N m^{2}C^{-2}
1 eV	1.6×10^{-19} J

7. Numerical Question Types – What to Expect

Exam Corner: Common Numerical Categories (2nd PUC)

1. **Distance of closest approach** given KE (in MeV) of α-particle and Z of target – convert MeV to Joules first.
2. **Radius/speed/energy at a given orbit** n for H or hydrogenic ions (He^+, Li^{2+}) – remember to use correct Z.
3. **Energy/wavelength/frequency of photon** emitted or absorbed during a transition between two given levels.
4. **Identifying the series** (Lyman/Balmer/etc.) from a given wavelength, or finding $\lambda_{max}/\lambda_{min}$ of a series.
5. **Ionisation energy / excitation energy** from a given state.
6. **Number of spectral lines** from a given excited state.
7. **Ratio-based questions**: ratio of radii/speeds/energies between two orbits or two hydrogenic species.
8. **Combined de Broglie + Bohr** numericals: find λ of electron in n^{th} orbit.

Common Mistakes to Avoid

- Forgetting to square Z in the energy formula (Z^2) but **not** squaring it in the radius formula (only n^2, single power of Z in denominator).
- Sign errors: E_n is negative, $PE_n = 2E_n$ (twice, same sign), $KE_n = -E_n$ (positive).
- Confusing **orbital revolution frequency** (ν_n, from T_n) with **photon frequency** (ν, from $h\nu = \Delta E$) – these are conceptually different quantities that happen to share a symbol.
- Not converting MeV $\rightarrow$ Joules ($\times 1.6 \times 10^{-13}$) before substituting into SI-unit formulas.
- Using $Z = 1$ by default even when the question specifies He^+ ($Z = 2$) or Li^{2+} ($Z = 3$).

D.P.P - I (PUC Theory)

1. Who discovered electron? (1M)
2. Name the series of hydrogen spectrum lying in ultraviolet and visible region. (1M)
3. What is Bohr's quantisation condition for the angular momentum of an electron in the second orbit? (1M)

Short Answer Questions (2 Marks)

1. Define Bohr's radius.
2. State the limitations of Bohr's atomic model.
3. Suppose you are given a chance to repeat the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14 K.) What results do you expect?
4. The ground state energy of hydrogen atom is $-13.6\,\text{eV}$. What are the kinetic and potential energies of the electron in this state?
5. If Bohr's quantisation postulate (angular momentum $= nh/2\pi$) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do we never speak of quantisation of orbits of planets around the sun?

Short Answer Questions (3 Marks)

1. The half-life period of a radioactive substance is 30 days. What is the time for $\frac{3}{4}$th of its original mass to disintegrate?
   Ans: $t = 60$ days.
2. How many α and β-particles are emitted when $^{232}_{90}\text{Th}$ changes to $^{208}_{82}\text{Pb}$?
   Ans: 6 α-particles and 4 β-particles.
3. Binding energies of $^{16}_{8}\text{O}$ and $^{35}_{17}\text{Cl}$ are 127.35 MeV and 289.3 MeV respectively. Which of the two nuclei are more stable?
   Ans: $^{35}_{17}\text{Cl}$ is more stable than $^{16}_{8}\text{O}$ (B.E./nucleon: 17.02 MeV vs 15.82 MeV).
4. What is the shortest wavelength present in the Paschen series of spectral lines?
   Ans: $\lambda = 818.9\,\text{nm}$.
5. A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom makes a transition from the upper level to the lower level?
   Ans: $\nu = 5.55 \times 10^{14}\,\text{Hz}$.
6. The radius of the innermost electron orbit of a hydrogen atom is $5.3 \times 10^{-11}\,\text{m}$. What are the radii of the $n = 2$ and $n = 3$ orbits?
   Ans: $r_2 = 2.12 \times 10^{-10}\,\text{m}$, $r_3 = 4.77 \times 10^{-10}\,\text{m}$.
7. In accordance with the Bohr's model, find the quantum number that characterises the earth's revolution around the sun in an orbit of radius $1.5 \times 10^{11}\,\text{m}$ with orbital speed $3 \times 10^4\,\text{m/s}$ (Mass of earth $= 6.0 \times 10^{24}\,\text{kg}$.)
   Ans: $n = 2.6 \times 10^{74}$.